

A study on the development of photoelastic experimental hybrid method for colour isochromatics (I)[†]

Jai-sug Hawong^{1,*}, Jeong-hwan Nam¹, Kyo-Hyoung Kim¹, O-sung Kwon², Gun Kwon² and Sung-han Park³

¹Department of Mechanical Engineering, Yeungnam University, Gyeongsan, 712-749, Korea

²Graduate School of Mechanical Engineering, Yeungnam University, Gyeongsan, 712-749, Korea

³Agency for Defense Development, Daejeon, 305-152, Korea

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Abstract

Isochromatics obtained from photoelastic experiment shows the stress distributions of the full field of a structure under load. Therefore, stress distributions of the structure can be read at a glance through isochromatics. Many experimental data can be obtained from isochromatics which are then used in various photoelastic experimental hybrid methods for stress analysis. Monochromatic light has however, until now been used in the photoelastic experimental hybrid method to produce black and white isochromatics. The use of black and white isochromatics in photoelastic experimental hybrid method for black and white isochromatics requires high fringe orders in order to obtain sufficient experimental data for photoelastic hybrid techniques. Accordingly, this paper develops the photoelastic experimental hybrid method for color isochromatics in which a fringe order of 1 is enough to gather the experimental data of the photoelastic experimental hybrid method. The method was applied to validate stress concentration problems. Experimental results from this study indicated that the photoelastic experimental hybrid method for color isochromatics is more precise than the photoelastic experimental hybrid method for black and white isochromatics. The use of few fringe orders in photoelastic experimental hybrid method for color isochromatics can offer significant advantages in stress analysis of real components using reflective-type photoelastic experimental method.

Keywords: Photoelastic experimental hybrid method for white and black isochromatics; Photoelastic experimental hybrid method for color isochromatics; White and black isochromatics; Color isochromatics; Monochromatic light; White light; Stress concentration factor

1. Introduction

A theoretical method and an experimental method are used to analyze the stresses of structures under load. The theoretical method includes a pure analytical method and a numerical method. The experimental method includes the strain gage method [1, 2], moiré method [3-5], photoelastic experimental method [7-9], fatigue [10, 11], SPATE [12, 13], J-integral [14, 15], etc.

Among them, the photoelastic experimental method (the full field method) has been used in a variety of ways.

A 2-dimensional photoelastic experimental method [16] and a 3-dimensional photoelastic experimental method [17, 18] exist.

Although the 2-dimensional photoelastic experimental method has been studied extensively, the 3-dimensional photoelastic experimental method has yet to be studied to such a

degree.

The transparent-type photoelastic method and the reflective-type photoelastic experimental method [19-22] have been applied to study 2-dimensional photoelastic experiment. However, the reflective-type dynamic photoelastic experimental method in the 2-dimensional photoelastic experiment has not been studied.

In the future, we will study this field. The difference between two principal stresses (σ_1 and σ_2) is represented by fringe orders in the photoelastic method. Therefore, separating the two principal stresses in the photoelastic method is important. The shear difference method [23], interferometer method [24], electrical-analogy method [25] and oblique incidence method [26] can be used to separate the two principal stresses in the photoelastic experiment. But in these methods, many problems have to be solved with precision and a lot of work has to be done to process experimental data. To overcome these difficulties, there have been many studies on the photoelastic experimental hybrid method, which is used to study the internal stress of a model or structures under arbitrary load [27-35].

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*Corresponding author. Tel.: +82 53 810 2445, Fax: +82 53 810 4627

E-mail address: jshawong@ynu.ac.kr

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Until now, fringe orders greater than 4 are needed to produce accurate results using the photoelastic experimental hybrid method in photoelastic experiment. However, in the reflective-type photoelastic experiment applied to real structures, fringe orders less than 4 are produced. Therefore, a photoelastic experimental hybrid method that can be used to analyze the stresses of a model producing fringe orders of less than 4 needs to be developed.

This research develops the photoelastic experimental hybrid method for color isochromatics for fringe orders of less than 4.

2. Basic theory

2.1 Complex function representation of stresses

Stress components are given by Eq. (1) using the Airy stress function [36] with Muskheliveli’s complex function [37].

$$\begin{aligned} \sigma_x &= \text{Re}\{2\phi'(z) - [\bar{z}\phi''(z) + \psi'(z)]\} \\ \sigma_y &= \text{Re}\{2\phi'(z) + [\bar{z}\phi''(z) + \psi'(z)]\} \\ \tau_{xy} &= \text{Im}\{\bar{z}\phi''(z) + \psi'(z)\} \end{aligned} \tag{1}$$

As shown in Eq. (1), once the stress functions $\phi(z)$ and $\psi(z)$ are determined, stress components (σ_x, σ_y and τ_{xy}) can be determined. Therefore, the main problem becomes the determination of $\phi(z)$ and $\psi(z)$ in the stress analysis. This research develops the method that can provide the stress functions $\phi(z)$ and $\psi(z)$ using the photoelastic experimental method. Eq. (2) is a conformal mapping function.

$$z = w(\zeta) \tag{2}$$

Substituting Eq. (2) into (1) and using the traction-free condition on a free surface, the relative equation between stress functions $\phi(z)$ and $\psi(z)$ is obtained as Eq. (3) [38].

$$\psi(\zeta) = -\frac{\bar{w}(\zeta)}{w'(\zeta)}\phi'(\zeta) - \bar{\phi}(\zeta) \tag{3}$$

Instead of obtaining two stress functions, $\phi(z)$ and $\psi(z)$, as in the conventional stress analysis, we obtain $\phi(\zeta)$ satisfying the boundary conditions.

Using Eq. (3), $\psi(\zeta)$ is obtained. Stress functions $\phi(\zeta)$ and $\psi(\zeta)$ can be transformed into $\phi(z)$ and $\psi(z)$ using the conformal mapping function. Substituting stress functions $\phi(z)$ and $\psi(z)$ into Eq. (1), stress components corresponding to the problem conditions are obtained.

What can be found by another method. Because $\phi(\zeta)$ is an analytic stress function, $\phi(\zeta)$ can be represented as a power series, as shown in Eq. (4).

$$\phi(\zeta) = \sum_{n=0}^N C_n(\zeta - \zeta_0)^n = \sum_{n=0}^N (a_n + ib_n)(\zeta - \zeta_0)^n \tag{4}$$

Complex coefficient C_n can be determined by using experimental data and a numerical method. Therefore, Eq. (4) is

determined.

Substituting Eq. (4) into Eq. (3) and using Eq. (3) with Eq. (4), Eqs. (2) and (1), stress components σ_x, σ_y and τ_{xy} are determined

$$\begin{aligned} \sigma_x &= \sum_{n=1}^N \text{Re}\{C_n[2f(n,\zeta) - g(n,\zeta)] + \bar{C}_n f(n,\zeta)\} \\ \sigma_y &= \sum_{n=1}^N \text{Re}\{C_n[2f(n,\zeta) + g(n,\zeta)] - \bar{C}_n f(n,\zeta)\} \\ \tau_{xy} &= \sum_{n=1}^N \text{Im}\{C_n g(n,\zeta) - \bar{C}_n f(n,\zeta)\} \end{aligned} \tag{5}$$

Eq. (5) are stress components determined from the above procedure.

Eq. (6) presents $f(n,\zeta)$ and $g(n,\zeta)$ of Eq. (5)

$$f(n,\zeta) \equiv \frac{n(\zeta - \zeta_0)^{n-1}}{\omega'(\zeta)} \tag{6a}$$

$$\begin{aligned} g(n,\zeta) &\equiv \left[\frac{\bar{\omega}(\zeta) - \bar{\omega}(\zeta_0)}{\omega'(\zeta)^2}\right]n(n-1)(\zeta - \zeta_0)^{n-2} \\ &\quad - \left[\frac{\bar{\omega}(\zeta) - \bar{\omega}(\zeta_0)}{\omega'(\zeta)^3}\omega''(\zeta) + \frac{\bar{\omega}(\zeta)}{\omega'(\zeta)^2}\right]n(\zeta - \zeta_0)^{n-1} \end{aligned} \tag{6b}$$

Substituting the stress components obtained from the above procedures into the stress optic law, that is, Eq. (7) [39], Eq. (8) is determined.

$$(\sigma_1 - \sigma_2)^2 = (\sigma_x - \sigma_y)^2 + (2\tau_{xy})^2 = \left(\frac{f \cdot N_f}{t}\right)^2 \tag{7}$$

$$\begin{aligned} \left(\frac{f \cdot N_f}{t}\right)^2 &= \left\{ \sum_{n=1}^N a_n \text{Re}[2f(n,\zeta) - 2g(n,\zeta)] \right. \\ &\quad \left. + \sum_{n=1}^N b_n \text{Im}[2f(n,\zeta) + 2g(n,\zeta)] \right\}^2 \\ &\quad + \left\{ \sum_{n=1}^N a_n \text{Im}[2g(n,\zeta) - 2f(n,\zeta)] \right. \\ &\quad \left. + \sum_{n=1}^N b_n \text{Re}[2f(n,\zeta) + 2g(n,\zeta)] \right\}^2 \end{aligned} \tag{8}$$

$f(n,\zeta)$ and $g(n,\zeta)$ are functions of the position coordinates.

Position coordinates are determined using the inverse mapping function ($\zeta = \omega^{-1}(z)$) when physical coordinates are known. Therefore Eq. (8) is a function of only a_n and b_n , and a_n and b_n are determined by using the nonlinear least square method.

Substituting the determined a_n and b_n into Eq. (4) and using Eqs. (2), (3) and (4) with the determined a_n and b_n , stress components [Eq. (1)] are determined. These procedures comprise the photoelastic experimental hybrid method.

2.2 Photoelastic experimental hybrid method for a circular hole

Eq. (9) is a conformal mapping function in the vicinity of a circular hole.

$$z = \omega(\zeta) = i\rho e^{-i\zeta} + z_1 = x + iy = r e^{i\theta} + z_1 \tag{9a}$$

$$\frac{r}{\rho} e^{i\theta} = i e^{-i\zeta} = e^{i(\frac{\pi}{2}-\zeta)} \tag{9b}$$

$\omega'(\zeta)$, $\omega(\zeta)$, $\zeta - \zeta_0$ and $\omega(\zeta)$ are determined using Eq. (9). Substituting the determined $\omega'(\zeta)$, $\omega(\zeta)$, $\zeta - \zeta_0$ and $\omega(\zeta)$ into Eq. (6), $f(n, r, \theta)$ and $g(n, r, \theta)$ for a circular hole are determined as shown in Eqs. (10a) and (10b), respectively.

$$f(n, r, \theta) = \frac{1}{r} e^{i(\frac{\pi}{2}-\theta)} n \left\{ \theta_0 - \theta + i \ln\left(\frac{r}{\rho}\right) \right\}^{n-1} \tag{10a}$$

$$g(n, r, \theta) = \frac{1}{r} \left(\frac{\rho^2}{r^2} - 1 \right) e^{-i3\theta} n(n-1) \left\{ \theta_0 - \theta + i \ln\left(\frac{r}{\rho}\right) \right\}^{n-2} + \frac{1}{r} \left(2\frac{\rho^2}{r^2} - 1 \right) e^{i(\frac{\pi}{2}-3\theta)} n \left\{ \theta_0 - \theta + i \ln\left(\frac{r}{\rho}\right) \right\}^{n-1} \tag{10b}$$

Substituting Eqs. (10a) and (10b) into Eq. (5) instead of $f(n, \zeta)$ and $g(n, \zeta)$, stress components are determined as Eq. (11). Substituting Eq. (11) for (7), Eq. (12) is determined.

$$\begin{aligned} \sigma_x(r, \theta) &= \sum_{n=1}^N \text{Re} \left\{ C_n [2f(n, r, \theta) - g(n, r, \theta)] + \overline{C_n} f(n, r, \theta) \right\} \\ \sigma_y(r, \theta) &= \sum_{n=1}^N \text{Re} \left\{ C_n [2f(n, r, \theta) + g(n, r, \theta)] - \overline{C_n} f(n, r, \theta) \right\} \\ \tau_{xy}(r, \theta) &= \sum_{n=1}^N \text{Im} \left\{ C_n g(n, r, \theta) - \overline{C_n} f(n, r, \theta) \right\} \end{aligned} \tag{11}$$

Eq. (12) is the stress optic law, which substitutes the position vector of the z-plane with the circular diameter p .

$$\begin{aligned} &= \left\{ \sum_{n=1}^N a_n \text{Re} [2f(n, r, \theta) - 2g(n, r, \theta)] \right. \\ &+ \left. \sum_{n=1}^N b_n \text{Im} [2f(n, r, \theta) + 2g(n, r, \theta)] \right\}^2 \\ &+ \left\{ \sum_{n=1}^N a_n \text{Im} [2g(n, r, \theta) - 2f(n, r, \theta)] \right. \\ &+ \left. \sum_{n=1}^N b_n \text{Re} [2f(n, r, \theta) + 2g(n, r, \theta)] \right\}^2 \\ &- \left(\frac{f \cdot N_f}{t} \right)^2 = 0 \end{aligned} \tag{12}$$

Applying the numerical method to Eq. (12), in which the

Table 1. Color isochromatic fringe characteristics.

Color	Fringe order (N)
Black	0
Gray	0.08
White	0.45
Pale yellow	0.6
Orange	0.8
Dull red	0.9
Purple (tint of passage no. 1)	1
Deep blue	1.08
Blue-green	1.22
Green-yellow	1.39
Orange	1.63
Rose red	1.82
Purple (tint of passage no. 2)	2
Green	2.35
Green-yellow	2.5
Red	2.65
Red/green transition (tint of passage no. 3)	3
Green	3.1
Pink	3.65
Pink/green transition (tint of passage no. 4)	4
Green	4.15

fringe orders and the position vector corresponding to the fringe orders are substituted, coefficients a_n and b_n of stress function are determined. Substituting the determined a_n and b_n for $\phi(\zeta)$ and using Eqs. (2) and (3), stress functions $\phi(z)$ and $\psi(z)$ are determined. Substituting the determined $\phi(z)$ and $\psi(z)$ into Eq. (1), the stress components in the vicinity of the circular hole in the plate under arbitrary load are determined. These procedures comprise the photoelastic experimental hybrid method. Until now, the needed experimental data measured from the black and white isochromatics have been applied to Eq. (12). The procedures obtaining $(\sigma_x, \sigma_y, \tau_{xy})$ using black and white isochromatics comprise the photoelastic experimental hybrid method for white and black isochromatics.

In this paper, the needed experimental data measured from color isochromatics are applied to the modified Eq. (12) and using the procedures explained in the previous page, stress components are determined. These procedures comprise the photoelastic experimental hybrid method for color isochromatics.

3. Experiment and experimental method

Both the transparent-type photoelastic experimental method and the reflective-type experimental method have been utilized in stress analysis. In general, the fringe orders of isochromatics are high in the transparent-type photoelastic experiment and very low in the reflective-type photoelastic experiment.

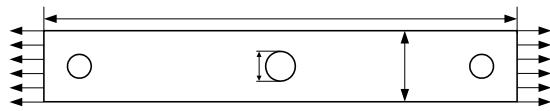
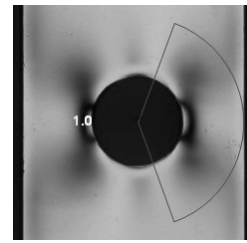
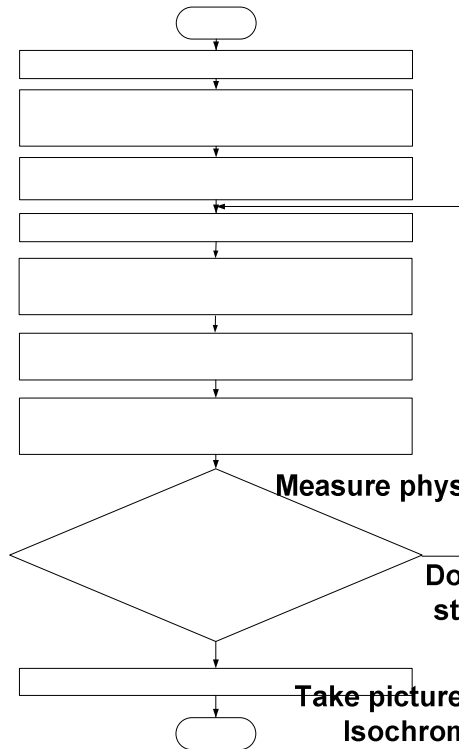
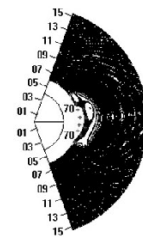


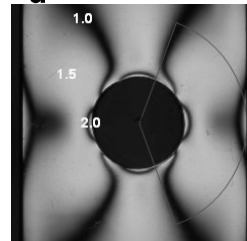
Fig. 1. Specimens of transmission type photoelastic experiment.



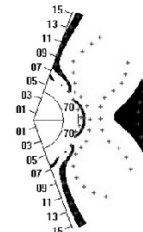
(a.1) Actual isochromatic fringe pattern ($\sigma_0 = 0.98MPa, N_f = 1$)



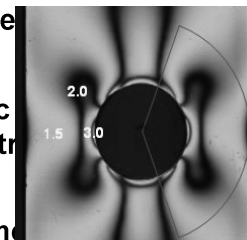
(a.2) Plotted isochromatic fringe pattern ($\sigma_0 = 0.98MPa, N_f = 1$)



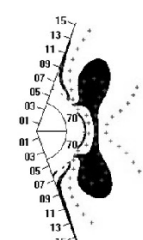
(b.1) Actual isochromatic fringe pattern ($\sigma_0 = 1.78MPa, N_f = 2$)



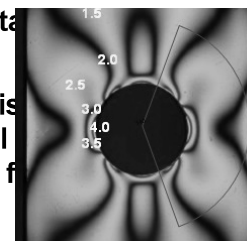
(b.2) Plotted isochromatic fringe pattern ($\sigma_0 = 1.78MPa, N_f = 2$)



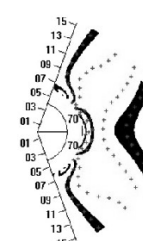
(c.1) Actual isochromatic fringe pattern ($\sigma_0 = 2.49MPa, N_f = 3$)



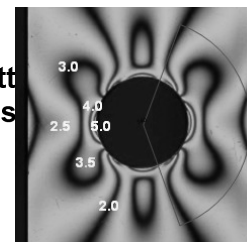
(c.2) Plotted isochromatic fringe pattern ($\sigma_0 = 2.49MPa, N_f = 3$)



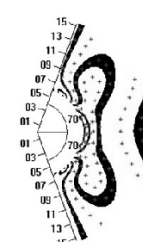
(d.1) Actual isochromatic fringe pattern ($\sigma_0 = 3.41MPa, N_f = 4$)



(d.2) Plotted isochromatic fringe pattern ($\sigma_0 = 3.41MPa, N_f = 4$)



(e.1) Actual isochromatic fringe pattern ($\sigma_0 = 4.06MPa, N_f = 5$)



(e.2) Plotted isochromatic fringe pattern ($\sigma_0 = 4.06MPa, N_f = 5$)

Fig. 2. Flow chart of photoelastic experimental hybrid method for color isochromatics.

To develop the photoelastic experimental hybrid method for color isochromatics, the transparent-type experimental device with a white light source is used in this research. Using a white light source, color isochromatics are produced. Table 1 indicates the relationship between fringe order (N) and color [40].

Color fringe orders are repeated according to the color sequence as shown in Table 1, but the tones of color isochromatics become more and more indistinct. Therefore, when fringe orders are greater than 4, the color isochromatic is vague. As shown in Table 1, when the fringe orders are greater than 4, we cannot distinguish fringe orders in detail.

To verify the effectiveness of the photoelastic experimental hybrid method for the color isochromatics developed in this research, the transparent-type photoelastic experiment was carried out on the rectangular specimen with a circular hole under uniform tension. Fig. 1 shows the specimen size used in this research. The height (h), the diameter (d), the width (w) and the thickness (t) of the finite rectangular plate are, respectively, h=200mm, d=12mm, w=30mm, and t=3mm.

The numerical method used in the photoelastic experimental hybrid method for color isochromatics is the Hooke and Jeeve's numerical method [41]. Fig. 2 shows the flow chart of

Fig. 3. Black and white isochromatic fringe patterns for rectangular sheet with a circular hole (d/w=0.39).

Yes

Plot stress components

END

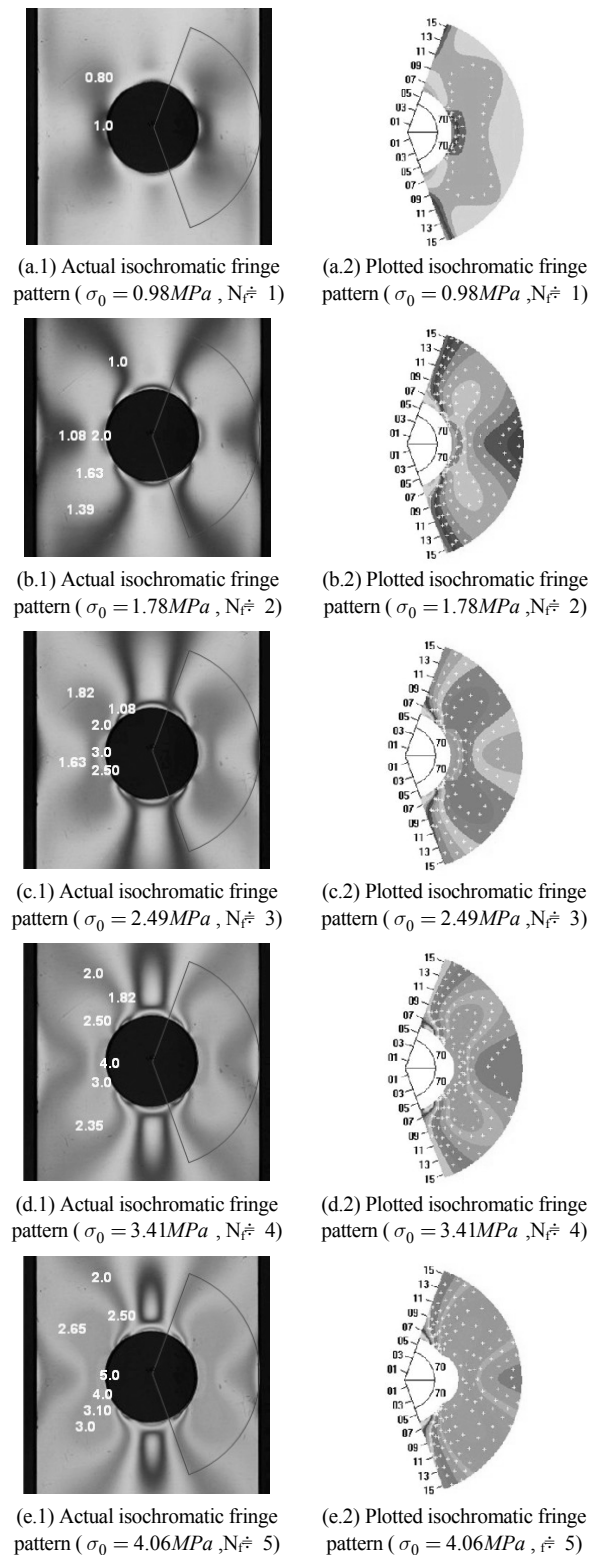


Fig. 4. Color isochromatic fringe patterns for rectangular sheet with circular hole ($d/w=0.39$).

4. Experimental results and discussions

(a), (b), (c), (d) and (e) in Fig. 3 show the actual black and white fringe patterns (left side) and plotted isochromatic fringe

patterns (right side) of specimens with applied loads of 146.5N, 266.64N, 393.29N, 511.05N and 608.82N, and fringe orders of about 1, 2, 3, 4 and 5 respectively, with d/w of 0.39.

Fig. 4 shows the color isochromatic fringe patterns obtained from the same loading and geometrical conditions as those of Fig. 3.

Plotted isochromatic fringe patterns (right side) of Fig. 3 and Fig. 4 show, respectively, graphic fan-shaped isochromatics obtained from the conventional photoelastic experimental hybrid method for black and white isochromatics and the photoelastic experimental hybrid method for color isochromatics developed in this research.

The numbers on the fringes indicate fringe orders. The fan-shaped region is the region where experimental data is measured. “+” marks on the fringes of the plotted isochromatics in Fig. 3 and Fig. 4 indicate the positions from where experimental data were measured.

Experimental data were measured on the centerline of the white and black bands in the black and white isochromatics. All the “+” marks are located on the centerline of the white and black bands in Fig. 3. This means that the conventional photoelastic experimental hybrid method is valid.

On the other hand, experimental data were measured on the centerline of each fringe order in the color isochromatics.

Since all the “+” marks are located on the centerline of each fringe order in Fig. 4, it means that the experimental hybrid method for the color isochromatics is effective.

(a.1), (b.1), (c.1), (d.1) and (e.1) of Fig. 3 and Fig. 4 show black and white isochromatics and color isochromatics for fringe orders of about 1, 2, 3, 4, and 5, respectively.

(a.2), (b.2), (c.2), (d.2) and (e.2) of Fig. 3 and Fig. 4 show the graphic isochromatics obtained from the photoelastic experimental hybrid method for black and white isochromatics and photoelastic experimental hybrid method for color isochromatics when the fringe order is about 1, 2, 3, 4 and 5.

When the maximum fringe order is 1, the graphic isochromatics obtained from the photoelastic experimental hybrid method for black and white isochromatics is completely different from the actual isochromatics, but the graphic isochromatics obtained from the photoelastic experimental hybrid method for color isochromatics is very similar to the actual isochromatics. When the maximum fringe order is about 2, the graphic isochromatics obtained from the photoelastic experimental hybrid method for black and white isochromatics is different from the actual black and white isochromatics, but graphic isochromatics obtained from the photoelastic experimental hybrid method for color isochromatics is almost identical to the actual isochromatics.

When the maximum fringe order is about 3, the graphic isochromatics obtained from the photoelastic experimental hybrid method for black and white isochromatics is very similar to the actual black and white isochromatics, and the graphic isochromatics obtained from the photoelastic experimental hybrid method for color isochromatics is identical to the actual color isochromatics.

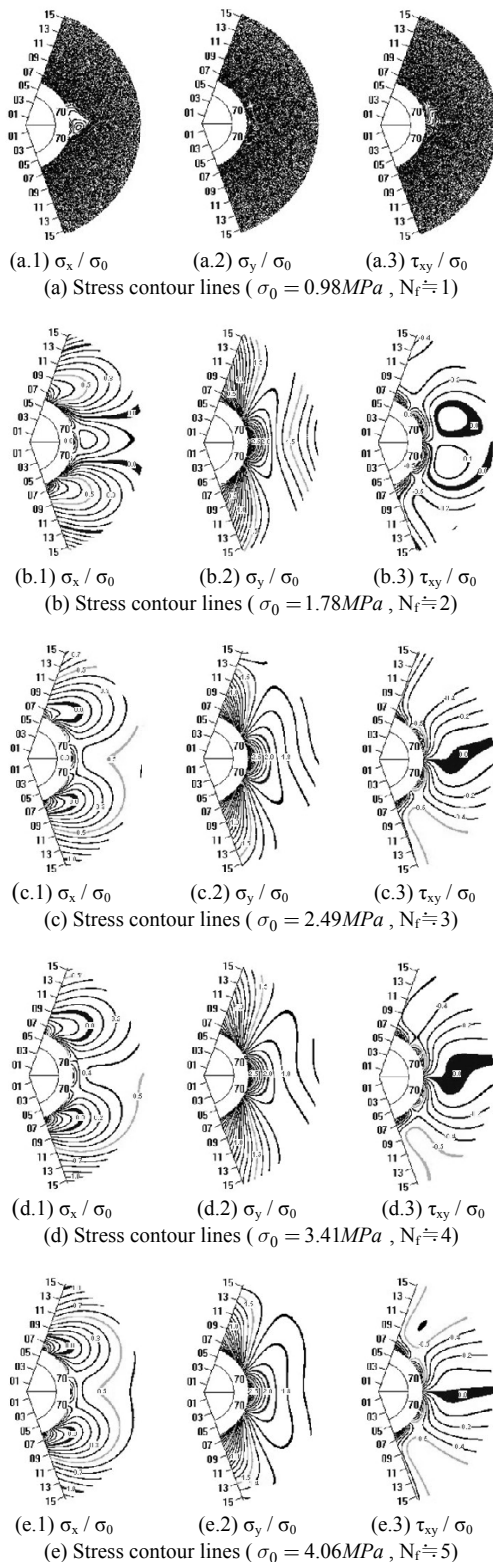


Fig. 5. Stress contour lines for rectangular sheet with circular hole ($d/w=0.39$).

When the maximum fringe order is about 4, the graphic isochromatics obtained from the photoelastic experimental hybrid method for black and white isochromatics is very simi-

lar to the actual black and white isochromatics, and the graphic isochromatics obtained from the photoelastic experimental hybrid method for color isochromatics is identical to the actual color isochromatics.

When the maximum fringe order is 5, the graphic isochromatics obtained from the photoelastic experimental hybrid method for black and white isochromatics is almost identical to the actual black and white isochromatics, and the graphic isochromatics obtained from the photoelastic experimental hybrid method for color isochromatics is completely identical to the actual color isochromatics.

The photoelastic experimental hybrid method for black and white isochromatics can be accurately used when the maximum fringe order is greater than or equal to 4.

The photoelastic experimental hybrid method for color isochromatics can be effectively used when the maximum fringe order is greater than 1 or equal to 1.

Fig. 5 and Fig. 6 are, respectively, stress contours obtained from the photoelastic experimental hybrid method for black and white isochromatics and for color isochromatics.

Fig. 5 shows the stress contours obtained from the photoelastic experimental hybrid method for black and white isochromatics of Fig. 3.

As shown in Fig. 3(a.1), when the maximum fringe order is about 1, black and white isochromatics are very vague and the position of the fringe orders is not clear; therefore, the needed experimental data cannot be measured, and thus, σ_x/σ_0 , σ_y/σ_0 , τ_{xy}/σ_0 cannot be calculated. They are almost zero, as shown in Fig. 5(a.1).

When the maximum fringe order is greater than about 3 and 4, a similar pattern of stress distributions of the stress components (σ_x/σ_0 , σ_y/σ_0 , τ_{xy}/σ_0) are obtained.

When the maximum fringe order is 5, the stress distributions of the stress components from the photoelastic experimental hybrid method for black and white isochromatics are very similar to those from the photoelastic experimental hybrid method for color isochromatics.

Fig. 6 shows the stress contours of stress components obtained from the photoelastic experimental hybrid method for color isochromatics of Fig. 4.

When the maximum fringe order is about 1, stress components are obtained. When the maximum fringe order is about 2, more accurate stress components are obtained.

When the maximum fringe order is greater than or equal to about 3, similar distribution patterns of accurate stress components are obtained.

As shown in Fig. 3, Fig. 4, Fig 5 and Fig. 6, the photoelastic experimental hybrid method for black and white isochromatics can be effectively used when the maximum fringe order is greater than or equal to 4.

The photoelastic experimental hybrid method for color isochromatics developed in this research can be effectively used when the maximum fringe order is greater than or equal to 1.

Table 2 gives the stress concentration factors α_k obtained

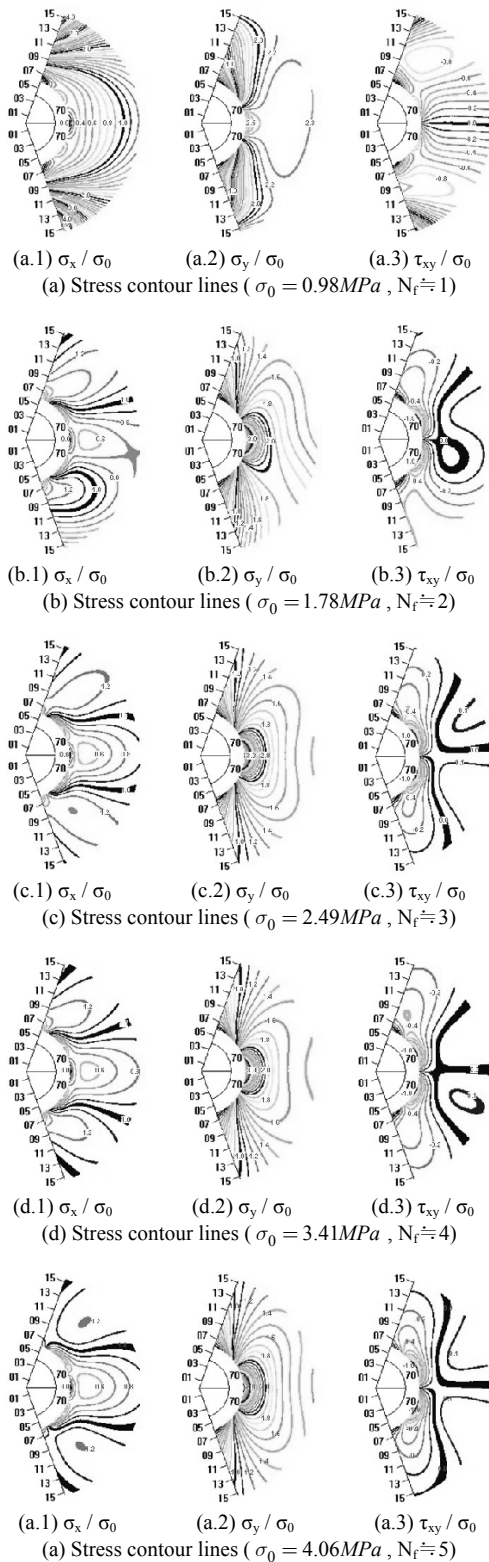


Fig. 6. Stress contour lines for rectangular sheet with circular hole (d/w=0.39).

from the photoelastic experimental hybrid method for black and white isochromatics and for color isochromatics for a rectangular plate with a circular hole under uniform tension.

Table 2. Values of stress concentration factors α_k for rectangular sheet with circular hole.

d/w	N	P(N)	σ_0	Stress Concentration Factor (α_k)				Theo value (41)
				Black & White		Color		
				Exp. value	Error [%]	Exp. value	Error [%]	
0.39	1	146.650	0.714	5.561	50.987	3.126	15.131	3.683
	2	266.636	1.298	4.622	25.471	3.790	2.882	
	3	373.290	1.817	4.470	21.352	3.755	1.937	
	4	511.052	2.487	4.305	16.867	3.729	1.239	
	5	608.818	2.963	4.079	10.734	3.714	0.831	

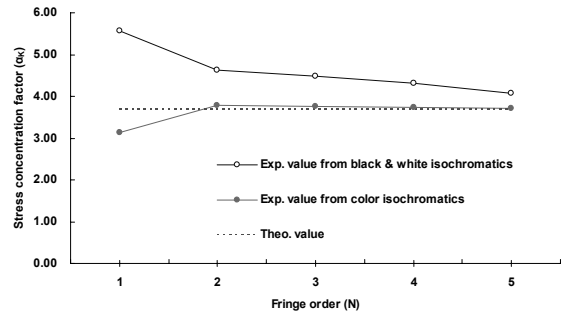


Fig. 7. Stress concentration factor α_k with respect to fringe order for a rectangular sheet with a circular hole.

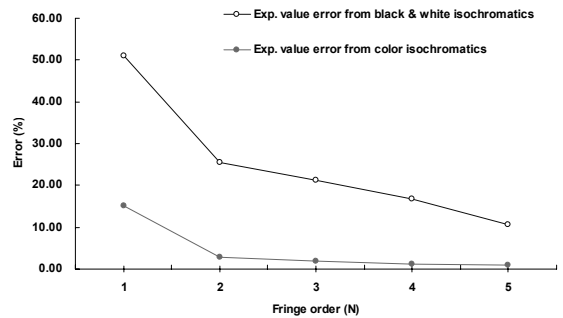


Fig. 8. Errors with respect to fringe order for rectangular sheet with circular hole.

$$Error(\%) = \frac{|Theo.value - Exp.value|}{Theo.value} \times 100$$

Fig. 7 and Fig. 8 respectively show stress concentration factors and errors with fringe orders.

As shown in Table 2, Fig. 7 and Fig. 8, when the maximum fringe order is 4, the error of experimental values from the photoelastic experimental hybrid method for black and white isochromatics is about 17%.

When the maximum fringe order is 5, the error from the photoelastic experimental hybrid method for black and white isochromatics is about 11%.

When the maximum fringe order is 2, the error from the photoelastic experimental hybrid method for color isochromatics is about 3%. When the maximum fringe order is 1, the errors from the photoelastic experimental hybrid method for

color isochromatics are about 15%.

The errors in the specific number of the fringe numbers in this paper universally resulted from the different geometries of mechanical elements.

As shown in Table 2, the photoelastic experimental hybrid method for black and white isochromatics can be effectively used in stress analysis when the maximum fringe orders are greater than or equal to about 4.

The photoelastic experimental hybrid method for color isochromatics can be effectively used in stress analysis when the maximum fringe order is greater than or equal to 1.

5. Conclusions

The following conclusions are obtained from previous experiments and experimental results.

1. A photoelastic experimental hybrid method for color isochromatics was developed and its validity was certified.
2. In stress analysis and separation of stress components using photoelastic technique, the photoelastic experimental hybrid method for black and white isochromatics can be effectively used when the maximum fringe order is greater than or equal to 4. However, when the maximum fringe order is greater than or equal to about 1, the photoelastic experimental hybrid method for color isochromatics can be effectively used.
3. When the maximum fringe orders are about 4 and about 5, the errors of experimental values obtained from the photoelastic experimental hybrid method for black and white isochromatics are about 17% and 11%, respectively. On the other hand, when maximum fringe orders are about 1 and about 2, the errors from photoelastic experimental hybrid color isochromatics are about 15% and about 3%, respectively, in stress concentration problems.

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Jai-Sug Hawong received a B.S. degree in Mechanical Engineering from Yeungnam University in 1974. Then he received his M.S. degree and Ph.D. degree from Yeungnam University in Korea in 1976 and from Kanto Gakuin University in Japan in 1990, respectively.

Prof. Hawong is currently a professor at the school of Mechanical Engineering at Yeungnam University, in Gyeongsan city, Korea. He is currently serving as President of Korea Society Mechanical Engineering. Prof. Hawong's research interests include static and dynamic fracture mechanics, stress analysis, experimental mechanics for stress analysis and composite material, etc.



Jeong-hwan Nam received a B.S. degree in Mechanical Engineering from Yeungnam University in 1986. Then he received his M.S. and Ph.D. from Yeungnam University in Korea in 1996 and from Saitama Institute of Technology in Japan in 2005, respectively. Dr. Nam is currently a

researcher at the school of Mechanical Engineering at Yeungnam University, in Gyeongsan city, Korea. Dr. Nam's research interests include mechanical design, stress analysis and experimental mechanics for stress analysis, etc.